CHAPTER 7 PRACTICE EXERCISES (*OPTIONAL)

7-01 LINES

2. $\theta = 120^{\circ}$

5. y = x + 3

6. 2x - 3y + 5 = 0

3. (-2, 1) and (0, 4)

4. (3, 2) and (5, -2)

7-01 Lines	
Find the slope of the line with the given inclination.	10. $x + 2y + 1 = 0$ and $-3x - 2y + 1 = 0$
	Find the distance from the point to the line.
	11. $x - y = 1$ and (1, 3)
60°	12. $2x + y - 2 = 0$ and $(-2, 1)$
	13. $y = \frac{1}{3}x - 1$ and (0, 3)
/	14. $y = -\frac{2}{5}x + \frac{3}{5}$ and $(-1, -2)$
2. $\theta = 120^{\circ}$ Find the inclination of the line that goes through the points.	Find the area of the triangle by (a) graphing the three vertices (b) find the equation of side AB (c) find the altitude
3. (-2, 1) and (0, 4)	to side AB and (d) calculate the area of the triangle.
4. (3, 2) and (5, -2)	15. A(0, 2), B(1, 3), C(-2, 5)
Find the inclination of the line with the given equation.	Find the equation of the line with the given inclination and <i>x</i> -intercept.
5. $y = x + 3$	16. $\theta = 30^{\circ}$ and (2, 0)
6. $2x - 3y + 5 = 0$	Mixed Review: Let $m=2\left(\cosrac{\pi}{3}+i\sinrac{\pi}{3} ight)$ and
Find the angle between the two given lines.	$n=4\left(\cosrac{5\pi}{3}+i\sinrac{5\pi}{3} ight).$
7. $y=rac{1}{2}x-3$ and $y=-rac{2}{3}x+1$	17. (6-07) Find <i>m</i> ² .
8. $y=-x-rac{1}{2}$ and $y=rac{3}{2}x+rac{1}{2}$	18. (6-07) Find $\frac{n}{m}$.
9. $2x - y = 0$ and $x + y = 2$	19. (6-06) Write m and n in standard form.
	20. (6-05) Find $\langle 1, \sqrt{3} \rangle \cdot \langle 2, -2\sqrt{3} \rangle$

7-02 Parabolas

 Given the focus and directrix, (a) how do you find the vertex? and (b) how do you know which way the parabola opens? 	$7. y^2 - 4x = 0$
Find the vertex, focus, and directrix of the following	8. $(x+2)^2 = -6(y-1)$
equations.	$9. y^2 - 4x + 8y + 12 = 0$
2. $x^2 = -8y$	Write the standard equation for the parabola with the
3. $3y^2 + 16x = 0$	following properties.
	10. Focus: $(5, 0)$, Directrix: $x = -5$
4. $(x-1)^2 = -16y$	11. Vertex: $(0, 0)$, Directrix: $y = 6$
$5. y^2 - 12x - 6y - 15 = 0$	
Graph the parabolas.	12. Focus: (1, 4), Vertex: (1, 7)
Graph the parabolas.	13. Focus: $(-2, 5)$, Directrix: $x = 1$
$6. x^2 = -2y$	Problem Solving

14. A thrown ball's path can be modeled by $y = -\frac{1}{40}x^2 + x$ where Mixed Review x is the horizontal distance in feet and y is the vertical distance

16. (7-01) Find the inclination of the line that goes through (2, 1) in feet from the point where the ball was thrown. What is the and (5, -3).

- 15. The Tyne Bridge in northeast England links Newcastle upon Tyne with Gateshead. It consists of two parabolic arches with a roadway between. If the arch can be $\frac{1}{2} + \frac{1}{2} + \frac{27}{27}$ where τ is $\frac{1}{2} + \frac{27}{27}$ where τ is modeled by $y = -\frac{1}{125}x^2 + \frac{27}{20}x$ where x is Bridge. (pixabay/ Michaela Wenzler) the horizontal distance in meters and y is the vertical distance in meters, find the height of the arch.
- 17. (7-01) Find the distance from P(0, 5) to the line y = 2x 4.

18. (6-05) Evaluate (2, -4) · (-5, 3).

and vertices (-2, 8) and (-2, -4).

vertical standard equation.

Mixed Review

(3, 0) and directrix x = 0.

19. (6-05) Evaluate (2, 0) · (0, -4).

0.

1 = 0.

19. (4-02) Evaluate the six trigonometric functions for $\theta = \frac{5\pi}{6}$.

20. (4-02) Evaluate the six trigonometric functions for $\theta = \frac{7\pi}{4}$.

7-03 Ellipses and Circles

highest point of the ball's path?

1. What is the difference between vertices and covertices?

Find the center, vertices, covertices, and foci of the following Problem Solving ellipses.

2. $\frac{x^2}{625} + \frac{y^2}{49} = 1$

 $3.\ 36x^2 + 16y^2 - 576 = 0$

4. $\frac{(x-3)^2}{9} + \frac{(y+2)^2}{36} = 1$

5. $12x^2 + 5y^2 + 48x + 10y - 7 = 0$

Find the standard equation of the ellipse with the following properties.

6. Vertices: (0, ±5), Foci: (0, ±1)

- 7. Vertices: (5, 1) and (-7, 1), Covertices: (-1, 4) and (-1, -2)
- 8. Foci: (2, 4) and (2, 0), Covertices: (5, 2) and (-1, 2)

Sketch the graph of the following ellipses.

9. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

10. $x^2 + 9y^2 - 18y = 0$

11. $16x^2 + 9y^2 - 64x + 18y - 71 = 0$

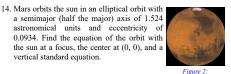
Eccentricity

12. Find the eccentricity of the ellipse $\frac{(x+5)^2}{160} + \frac{y^2}{144} = 1$.

13. Find the standard equation of the ellipse with eccentricity of $\frac{2}{3}$ 20. (6-03) Evaluate (2, 0) + (0, -4) both graphically and algebraically.

7-04 Hyperbolas

1. What is the difference between the transverse axis and the conjugate axis? How are they related to the major and minor	2. $\frac{y^2}{64} - \frac{x^2}{16} = 1$
axes of an ellipse?	$3.\ 20x^2 - 25y^2 - 200 = 0$
Find the center, vertices, asymptotes, and foci of the following hyperbolas.	4. $\frac{(x+6)^2}{9} - \frac{(y-3)^2}{8} = 1$



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(wikimedia/NASA/
                                                   JPL/USGS)
15. An equation that models the moon's orbit
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around
                             the
                                                    earth
                                              = 1. Find
                                                                            the
\frac{1.4803 \times 10^{11}}{1.4758 \times 10^{11}} + \frac{1.4758 \times 10^{11}}{1.4758 \times 10^{11}}
eccentricity of the moon's orbit.
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16. (7-02) Find the vertex, focus, and directrix of $y^2 - 6x - 6y - 3 =$

17. (7-02) Write the standard equation for the parabola with vertex

18. (7-01) Find the distance from the point (5, 1) to the line 2x - y + y



JPL)

5. $100x^2 - 81y^2 + 200x + 648y + 6904 = 0$

base is (112.5, 0).

Find the standard equation of the hyperbola with the following properties.

6. Foci: (±4, 0), Vertices: (±2, 0)

7. Vertices: (3, 4) and (3, 10), Covertices: (-1, 7) and (7, 7)

8. Asymptotes: $y = 3 \pm \frac{2}{2}(x-1)$, Vertex: (1, 7)

Sketch the graph of the following hyperbolas.

9. $\frac{x^2}{9} - \frac{y^2}{10} = 1$

10. $x^2 - y^2 - 6x + 4y + 9 = 0$

11. $4x^2 - v^2 + 8x + 8v - 16 = 0$

Eccentricity

- 12. Find the standard equation of the hyperbola with $e = \frac{4}{2}$ and vertices $(\pm 3, 0)$.
- 13. What is the eccentricity of $16x^2 9y^2 + 96x + 36y 36 = 0$?

Problem Solving

14. The cooling tower at the electrical power generating station in 19. (7-01) Find the angle between y = -4x and 2x - y + 3 = 0. Michigan City, Indiana, is modeled by a hyperbola and is about 300 feet tall. Write a model for the sides of the tower if the 20. (2-04) Divide using synthetic division: $(2x^3 + 4x^2 - x + 3) \div (x^3 + 4x^2 - x + 3)$ center is at (0, 200), the vertices are (± 75 , 200), and point at the +1).

7-05 ROTATED CONICS

1. When you are using $\cot 2\theta = \frac{A-C}{B}$ and get a negative angle, Classify the conic and sketch its graph using a graphing utility. how do you get the correct positive angle? Do you do this before or after dividing by 2? 11. $x^2 + 2xy + y^2 + 2x = 0$ Classify the conic and rewrite it in standard form by 12. $2x^2 + 5xy + y^2 - 3x = 0$ eliminating the Bxy term. 13. $x^2 + 5xy + y^2 - 4y = 0$ 2. 2xy = -914. $x^2 - 2xy + 2y^2 + 2x - 3y = 0$ 3. $7x^2 + 4xy + 7y^2 - 45 = 0$ 4. $x^2 - 2\sqrt{3}xy + 3y^2 + 8\sqrt{3}x + 8y = 0$ 15. $4x^2 - 4xy + y^2 - 5y - 7 = 0$ 5. $24x^2 - 2\sqrt{3}xy + 22y^2 - 525 = 0$ Mixed Review Classify the conic, rewrite it in standard form, and sketch its 16. (7-04) Sketch a graph of $\frac{y^2}{16} - \frac{x^2}{4} = 1$. graph. 17. (7-03) Sketch a graph of $\frac{x^2}{4} + \frac{y^2}{16} = 1$. 6. $7x^2 - 50xy + 7y^2 + 72 = 0$ 7. $x^2 + 4xy + 4y^2 - 4x + 2y = 0$ 18. (7-02) Sketch a graph of $x^2 = -12y$. 8. $6x^2 + 12xy + y^2 + 3 = 0$ 19. (7-01) Sketch a graph of a line with inclination of $\frac{\pi}{4}$ and goes through (0, 0). 9. $14x^2 + 15xy + 6y^2 - \frac{27}{2} = 0$ 20. (6-03) Evaluate (2, -3) + 2(0, 1). 10. $2x^2 + 3xy - 2y^2 - 10 = 0$



Klement)

shadow. The shadow falls on a scale to tell the time. The tip of the rod traces a hyperbola over the course of a day. This is Figure 5: Sundial in called the declination line. If a certain Cambridge, UK. (Richard Wright) declination line is modeled by $\frac{y^2}{4} - \frac{x^2}{100} = 1$, what is its eccentricity?

Mixed Review

16. (7-03) Find the center, vertices, covertices, and foci of $\frac{(x+1)^2}{16} + \frac{(y-4)^2}{9} = 1.$

15. A sundial is made of a rod that casts a

- 18. (7-02) Write the standard equation for the parabola with vertex (2, 3) and directrix y = -1.

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Graph the parametric equations.
 2. \begin{cases} x = 4t^2 \end{cases}
        \int u = 8t + 2
 3. \left\{ \begin{array}{l} x = 4t + 1 \\ y = 2t^2 \end{array} \right.
  4. \begin{cases} x = 3 \sec \theta \\ y = 2 \tan \theta \end{cases}
 5. \begin{cases} x = 2\sin\theta\\ y = 4\cos\theta \end{cases}
      Write a set of parametric equations for the following
      conditions.
 6. Parabola with vertex at (-2, -1) and focus (-5, -1).
 7. Horizontal ellipse with center (0, 0), vertices (\pm 5, 0), and foci
     (\pm 3, 0).
 8. y = 3x + 1
 9. (y-2)^2 = -16x
      Eliminate the parameter.
10. \begin{cases} x = 4t^2 \\ y = 8t + 2 \end{cases}
11. \begin{cases} x = 3t \\ y = \frac{9}{t} \end{cases}
12. \begin{cases} x = 7\sin\theta\\ y = 4\cos\theta \end{cases}
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7-07 Polar Coordinates

7-06 PARAMETRIC EOUATIONS

1. What is a parameter?

1. What does each part of the polar coordinate (r, θ) represent?	Convert the points from polar to rectangular or rectangular to polar.
Graph the polar coordinates.	1
2. $J\left(3,0 ight)$ and $K\left(4,rac{\pi}{3} ight)$	8. $(5, \frac{3\pi}{2})$
3. $M\left(-1,\frac{4\pi}{3}\right)$ and $N\left(-2,\frac{11\pi}{6}\right)$	9. $\left(3, \frac{5\pi}{6}\right)$
4. $Q(4, \frac{\pi}{2})$ and $R(-4, \frac{3\pi}{2})$	10. $(2\sqrt{2}, -2\sqrt{2})$
Find two other ways to write each coordinate.	11. $(-1, -\sqrt{3})$
	Convert the equations from polar to rectangular.
5. $(2, \frac{\pi}{4})$	12. $r = 5$
6. $\left(-3, \frac{2\pi}{3}\right)$	
$= \begin{pmatrix} 1 & 3\pi \end{pmatrix}$	13. $\theta = \frac{5\pi}{6}$
7. $(1, \frac{3\pi}{8})$	14. $r = 5 \csc \theta$

13. $\begin{cases} x = 6 \tan \theta \\ y = 5 \sec \theta \end{cases}$

Problem Solving

- 14. On televised baseball games, sometimes the distance of a home run is given, but it was not measured. The distance was calculated. Let $\left\{ x = 35t \right\}$ model the path of the ball $y=-16t^2+30t$ after it was hit. How far did the ball go? (Hint: Let v = 0 and find *t*.)
- 15. A solar oven cooks food by reflecting the sunlight off a parabolic mirror. The food is placed at the focus where all the light is focused. Write a set of parametric equations to model the surface of the mirror if the vertex is at (0, 0) and the focus is 2 feet Figure 6: Solar oven above the vertex. (wikimedia/John Hill)

Mixed Review

- 16. (7-05) Classify the conic and rewrite it in standard form by eliminating the Bxy term. $x^2 - 2xy + y^2 - x - y = 0$.
- 17. (7-04) Find the standard equation of the hyperbola with vertices $(4, \pm 3)$ and foci $(4, \pm 5)$.
- 18. (7-03) Find the center, vertices, covertices, and foci of the
- 20. (7-01) Find the distance from the point (2, 0) to the line 2x y + y $\dot{4} = 0.$

- 19. (7-02) Graph $(x-1)^2 = 4(y+2)$.

ellipse $\frac{(x+1)^2}{16} + \frac{y^2}{2} = 1.$

17. (7-03) Sketch the graph of $x^2 + 16y^2 + 4x - 12 = 0$.

15. $r = 4 \cos \theta$

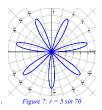
Mixed Review

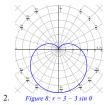
16. (7-06) Graph the parametric equations $\begin{cases} x = 3\cos\theta \\ y = 2\sin\theta \end{cases}$

17. (7-06) Eliminate the parameter of $\begin{cases} x = 3\cos\theta\\ y = 2\sin\theta \end{cases}$

7-08 Graphs of Polar Eduations

Identify the type of graph.





Identify the symmetry of the polar graphs.

3. $r = 3 - 4 \cos \theta$

4. $r = 4 \cos \theta$

5. $r = 1 + \sin \theta$

- Find the maximum values of |r| and the zeros of r.
- 6. $r = 2 + \cos \theta$

7-09 POLAR EOUATIONS OF CONICS

1. Review the lessons on ellipses and hyperbolas. What does a, b, $5. r = \frac{5}{1+4\cos\theta}$ and c stand for? And how do you use them to find e?

Identify the conic and sketch its graph.

2. $r = \frac{4}{2 + \sin \theta}$

3. $r = \frac{3}{1 - \cos \theta}$



- 18. (7-05) Classify the conic and rewrite it in standard form by eliminating the Bxy term. xy = 9
- 19. (7-04) Find the standard equation of the hyperbola with foci (0, ± 10) and vertices (0, ± 8).
- 20. (7-02) Find the vertex, focus, and directrix $x^2 = 4y$.
- 8. $r = 3 \cos 2\theta$ Graph the polar equations. 9. $r = 4 \cos \theta$
- $10 r = 4 \sin \theta$

7. $r = 4 \sin \theta$

- 11. $r = 2 + \cos \theta$
- 12. $r = 3 \cos 2\theta$
- 13 $r = 2 3 \sin \theta$

Use a graphing utility to graph the polar equations.

- 14. $r = 5 \sin 4\theta$
- 15. $r = 1 + \cos \theta$

Mixed Review

- 16. (7-07) Convert the point to rectangular coordinates: $(2, \frac{5\pi}{4})$
- 17. (7-07) Convert the equation to rectangular coordinates: $r = 2 \sin \theta$

18. (7-06) Graph
$$\begin{cases} x = 2t \\ y = \frac{1}{2}t^3 \end{cases}$$

19. (7-05) Classify the conic: $3x^2 - xy + 4y^2 - 12x + 3y + 2 = 0$.

20. (7-03) Find the standard equation of the ellipse with vertices (±5, 0) and foci (±3, 0).

- Write the polar equation of the conic with its focus at the pole and the given properties. 9. parabola with directrix x = 4
- 10. hyperbola with eccentricity e = 2 and directrix y = -2
- 11. ellipse with eccentricity $e = \frac{2}{3}$ and directrix y = 6
- 12. parabola with vertex $\left(4, \frac{3\pi}{2}\right)$
- 13. ellipse with vertices (7, 0) and $(3, \pi)$
- 14. hyperbola with vertices $(3, \pi)$ and (-5, 0)

Problem Solving

15. The comet Hale-Bopp has an elliptical orbit with eccentricity of

0.995. It has a semi-major axis of about 250 astronomical units. (a) Write an equation for the orbit of Hale-Bopp with the sun at one focus. (b) How close does the comet come to the sun?

Mixed Review

- 16. (7-08) Graph $r = 2 \sin \theta$.
- 17. (7-08) Identify the symmetry of $r = 3 \cos \theta$.
- 18. (7-07) Convert $r = 3 \cos \theta$ to rectangular coordinates.
- 19. (7-04) Find the standard equation of the hyperbola with foci (±10, 0) and vertices (±6, 0).

20. (7-01) Find the angle between the lines $y = \frac{2}{3}x + 1$ and $y = x - \frac{2}{3}x + 1$ 2

21. Find the polar equation for a hyperbola with the vertices $\left(2,\frac{\pi}{2}\right)$

7-Review

Ake this test as you would take a test in class. When you are finished, check your work against the answers. On this assignment round your answers to three decimal places unless otherwise directed. 1. Find the inclination of 2x + y - 3 = 0 in degrees. 12. Eliminate the parameter from $x = \sqrt{t}$ and $v = 2t^2$.

- 2. Find the angle between the lines 2x + y 3 = 0 and x 2y + 1 = 13. Convert $(4, \frac{\pi}{3})$ to rectangular coordinates. 14. Find another polar coordinate that represents $\left(4,\frac{\pi}{n}\right)$. 3. Find the distance between (2, 4) and 2x + y - 3 = 0.
- 15. Convert $r = 4 \sec \theta$ to rectangular form. 4. Classify the conic $4x^2 + 9y^2 - 8x + 36y + 4 = 0$.
 - 16. Graph the polar coordinate $\left(2, \frac{7\pi}{e}\right)$.

 $e = \frac{1}{2}$.

and $(-6, \frac{3\pi}{2})$.

- 17. Classify the graph of $r = \frac{6}{1-3\cos\theta}$.
- 7. Find the standard form of the equation of the parabola with 18. Find one focus of $r = \frac{6}{1-3\cos\theta}$ focus (3, 0) and directrix x = -1.
- 19. Classify the graph of $r = \frac{3}{1+\sin\theta}$ 8. Find the standard form of the hyperbola with vertices (2, 5) and (-4, 5) and b = 5. 20. Find the polar equation for an ellipse with directrix x = -6 and
- 9. Classify the conic $x^2 2xy + 2y^2 + 3x 5y + 12 = 0$.

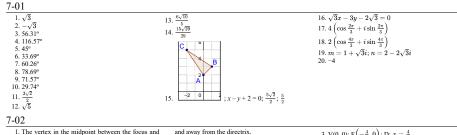
5. Find the foci of $4x^2 + 9y^2 - 8x + 36y + 4 = 0$.

6. Graph $4x^2 + 9y^2 - 8x + 36y + 4 = 0$.

10. What degree is $x^2 - 2xy + 2y^2 + 3x - 5y + 12 = 0$ rotated?

11. Graph the parametric equations $x = \sqrt{t}$ and $y = 2t^2$.

ANSWERS

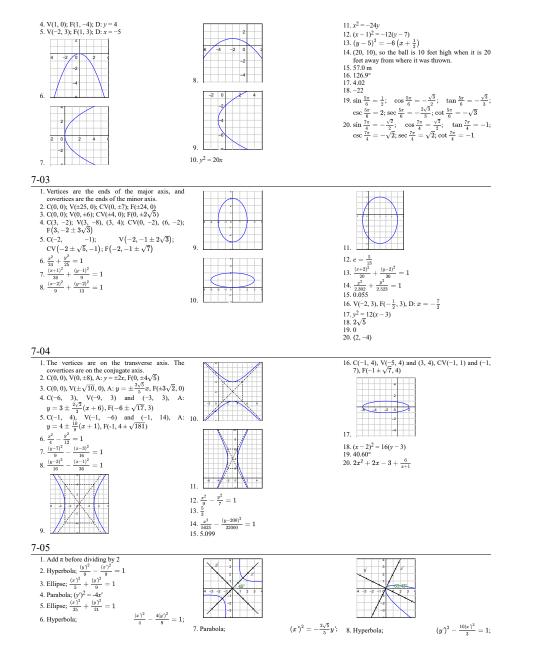


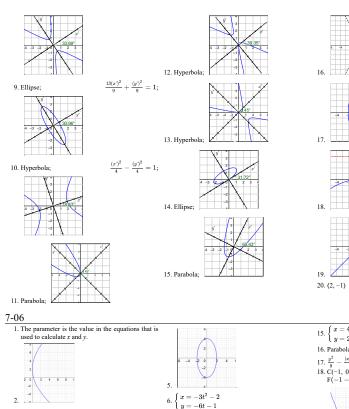
the directrix: The parabola opens around the focus 2, V(0, 0): F(0, -2): D: v = 2

3. V(0, 0); $F\left(-\frac{4}{3}, 0\right)$; D: $x = \frac{4}{3}$

6. $r = \frac{5}{2+3\cos\theta}$ Use a graphing utility to graph the polar equation. Identify the graph.







 $\int x = 5 \cos t$

 $y = 4 \sin t$

y = t + 1

 $\int x = -4t^2$

11. xy - 27 = 0 $12. \frac{x^2}{49} + \frac{y^2}{16} = 1$ $13. \frac{y^2}{25} - \frac{x^2}{36} = 1$

14. 65.625 feet

3.

9. y = -8t + 210. $(y - 2)^2 = 16x$

 $\int x = \frac{t}{3}$

7.

0 - - - 2 0

8 -6 -4) -2 0 2 4 6

1. r = distance from the origin; θ = angle

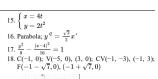
counterclockwise from the positive x-axis.

3

4.

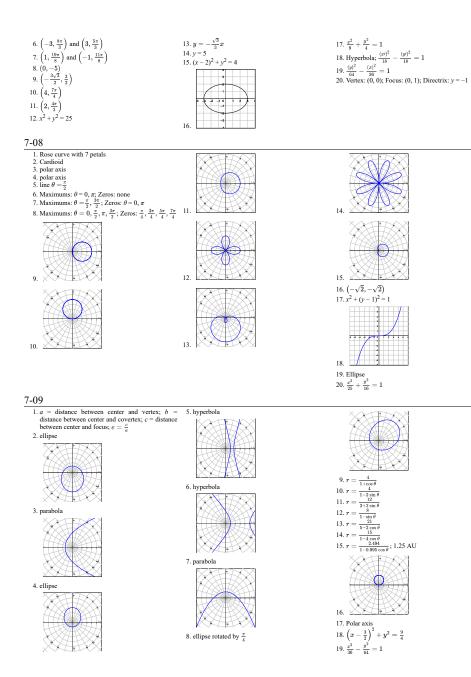
7-07

2. ×





4
5. $\left(2, \frac{9\pi}{4}\right)$ and $\left(-2, \frac{5\pi}{4}\right)$







6.

